# NEW STANDARD ACADEMY

**Test Type : Unit Test # 03**

Do not open this Test Booklet until you are asked to do so. **14-08-2023**

## JEE(MAIN): 12th Undergoing/Pass Students

Read carefully the Instructions on the Back Cover of this Test Booklet.

### Important Instructions :

1. Immediately fill in the form number on this page of the Test Booklet with Blue/Black Ball Point Pen. Use of pencil is strictly prohibited.
2. The candidates should not write their Form Number anywhere else (except in the specified space) on the Test Booklet/Answer Sheet.
3. The Test Booklet consists of 90 questions.
4. There are three parts in the question paper 1,2,3 consisting of Physics, Chemistry and Mathematics having 30 questions in each subject and each subject having Two sections. (i) Section-I contains 20 multiple choice questions with only one correct option. Marking scheme : +4 for correct answer, 0 if not attempted and –1 in all other cases. (ii) Section-II contains 10 Numerical Value Type questions. Attempt any 5 questions. First 5 attempted questions will be considered for marking. Marking scheme : +4 for correct answer, 0 if not attempted and –1 in all other cases.
5. Use Blue/Black Ball Point Pen only for writting particulars/marking responses on Side –1 and Side–2 of the Answer Sheet. Use of pencil is strictly prohibited.
6. No candidate is allowed to carry any textual material, printed or written, bits of papers, mobile phone any electronic device etc, except the Identity Card inside the examination hall/room.
7. Rough work is to be done on the space provided for this purpose in the Test Booklet only.
8. On completion of the test, the candidate must hand over the Answer Sheet to the invigilator on duty in the Room/ Hall. However, the candidate are allowed to take away this Test Booklet with them.

Name of the Candidate(In Capitals) _____

Date of Examintation _____

Candidate`s Signature:_____ Invigilator`s Signature: _____

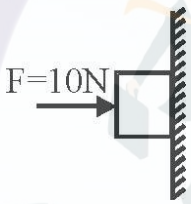
## PART-1 : PHYSICS

### SECTION-I : (Maximum Marks: 80)

This section contains **20 questions**. Each question has 4 options for correct answer. Multiple-Choice Questions (MCQs) **Only one option is correct**. For each question, marks will be awarded as follows:

*Full Marks* : +4 If correct answer is selected.
*Zero Marks* : 0 If none of the option is selected.
*Negative Marks* : –1 If wrong option is selected.

**1.** A horizontal force of 10N is necessary to just hold a block stationary against a wall. The coefficient of friction between block and wall is 0.2. The weight of block is:-



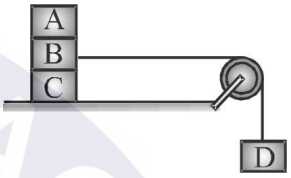

(A) 20 N (B) 50 N (C) 100 N (D) 2N

**2.** A 40 kg slab (B) rests on a smooth floor as shown in figure. A 10 kg block (A) rests on the top of the slab. The static coefficient of friction between slab and block is 0.6 while the kinetic friction coefficient is 0.4. The block (A) is acted upon by a horizontal force 100 N. If g = 9.8 $m/s^2$, the resulting acceleration of the slab (B) will be :-


(A) 0.98 $m/s^2$ (B) 1.47 $m/s^2$

(C) 1.52 $m/s^2$ (D) 6.1 $m/s^2$

**3.** These blocks A, B and C of equal mass m are placed one over the other on a smooth horizontal ground as shown in figure. Coefficient of friction between any two blocks of A, B and C is 1/2. The maximum value of mass of block D so that the blocks A, B and C move without slipping over each other is :-



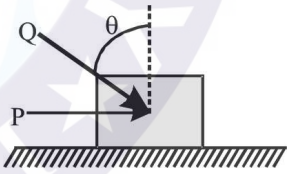

(A) 6m

(B) 5m

(C) 3m

(D) 4m

**4.** A block of mass m lying on a rough horizontal plane is acted upon by a horizontal force P and another force Q inclined at an angle θ to the vertical. The block will remain in equilibrium if the coefficient of friction between it and the surface is :-




(A) $\frac{P + Q\sin\theta}{mg + Q\cos\theta}$

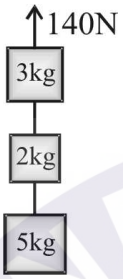
(B) $\frac{P\cos\theta + Q}{mg - Q\sin\theta}$

(C) $\frac{P + Q\cos\theta}{mg + Q\sin\theta}$

(D) $\frac{P\sin\theta - Q}{mg - Q\cos\theta}$

**Space for Rough Work**

**5.** What will be the acceleration of blocks and tension in the string connecting the blocks 2kg and 5kg for the given figure :-




(A) 14 m/s$^2$, 120 N

(B) 4m/s$^2$, 70 N

(C) 4 m/s$^2$, 30 N

(D) 14 m/s$^2$, 140 N

**6.** The total mass of an elevator with a 80 kg man in it is 1000 kg. This elevator moving upward with a speed of 8 m/sec, is brought to rest over a distance of 16m. The tension T in the cables supporting the elevator and the force exerted on the man by the elevator floor will respectively be :-

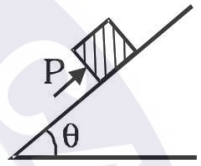
(A) 7800 N, 624 N

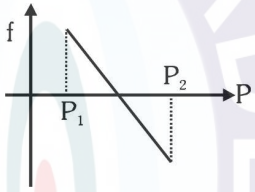
(B) 624 N, 7800 N

(C) 11800 N, 624 N

(D) 624 N, 78 N

**7.** A block of mass m is on an inclined plane of angle $\theta$. The coefficient of friction between the block and the plane is $\mu$ and $\tan\theta > \mu$. The block is held stationary by applying a force P parallel to the plane. The direction of force pointing up the plane is taken to be positive. As P is varied from $P_1 = mg(\sin\theta - \mu\cos\theta)$ to $P_{13} = mg(\sin\theta + \mu\cos\theta)$, the frictional force f versus P graph will look like




(A)
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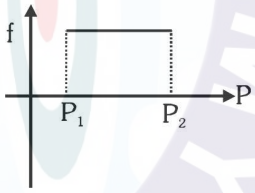
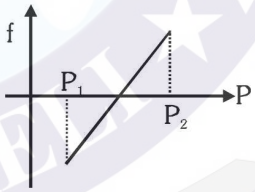



(C)
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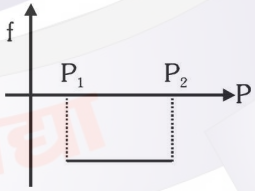



**Space for Rough Work**

**8.** Figure shows a 5 kg ladder hanging from a string that is connected with a ceiling and is having a spring balance connected in between. A man of mass 25 kg is climbing up the ladder at acceleration 1 $m/s^2$. Assuming the spring balance and the string to be massless, the reading of the spring balance is :-

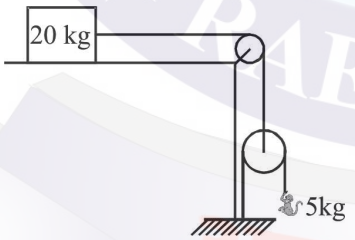
(A) 30 kg (B) 32.5 kg

(C) 35 kg (D) 37.5 kg

**9.** A heavy uniform chain lies on a horizontal table top. If the coefficient of friction between the chain and the table surface is 0.25, then the maximum fraction of the length of the chain that can hang over one edge of the table is-

(A) 20% (B) 25% (C) 35% (D) 15%

**10.** What will be the acceleration of 20 kg blocks. If the monkey climbs up the rope with acceleration of $2m/s^2$. If all surfaces are smooth string and pulley are light.:-




(A) $2\ m/s^2$ (B) $3\ m/s^2$ (C) $4\ m/s^2$ (D) 0

**11.** A projectile is projected with velocity $kv_e$ in vertically upward direction from the ground into the space. ($v_e$ is escape velocity and $k < 1$). If air resistance is considered to be negligible then the maximum height from the centre of earth to which it can go, will be : (R = radius of earth)

(A) $\frac{R}{k^2+1}$ (B) $\frac{R}{k^2-1}$

(C) $\frac{R}{1-k^2}$ (D) $\frac{R}{k+1}$

**12.** At a given place where acceleration due to gravity is 'g' $m/sec^2$, a sphere of lead of density 'd' $kg/m^3$ is gently released in a column of liquid of density '$\rho$' $kg/m^3$. If $d > \rho$, the sphere will:-

(A) Fall vertically with an acceleration 'g' $m/sec^2$

(B) Fall vertically with no acceleration

(C) Fall vertically with an acceleration $g\left(\frac{d-\rho}{d}\right)$

(D) Fall vertically with an acceleration $g\left(\frac{\rho}{d}\right)$

**13.** $V_e$ and $V_p$ denote the escape velocities from the earth and another planet having twice the radius and the same mean density as that for the earth. Then :-

(A) $V_e = V_p$

(B) $V_e = 2\,V_p$

(C) $V_e = V_p/2$

(D) $V_e = V_p / \sqrt{2}$

**Space for Rough Work**

**14.** An asteroid of mass m is approaching earth initially at a distance of 10 Re from surface of earth and speed $V_i$. It hits the earth with a speed $V_f$ (Re and Me are radius and mass of earth), then-

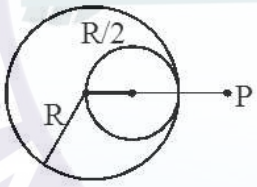
(A) $V_f^2 = V_i^2 + \frac{2Gm}{MeR}\left(1 - \frac{1}{10}\right)$

(B) $V_f^2 = V_i^2 + \frac{2GMe}{Re}\left(1 + \frac{1}{10}\right)$

(C) $V_f^2 = V_i^2 + \frac{2GMe}{Re}\left(1 - \frac{1}{10}\right)$

(D) $V_f^2 = V_i^2 + \frac{2GM}{Re}\left(1 - \frac{1}{10}\right)$

**15.** A solid sphere of uniform density and radius R applies a gravitational force of attraction equals to $F_1$ on a particle placed at P, distance 2R from the centre. O of the sphere. A spherical cavity of radius R/2 is now made in a sphere as shown in figure. The sphere with cavity now applies a gravitational force $F_2$ on the same particle placed at P. The ratio $F_2/F_1$ will be -




(A) 1/2 (B) 7/9 (C) 3 (D) 9/7

**16.** The ratio of the radius of a planet 'A' to that of planet 'B' is 'r'. The ratio of acceleration due to gravity on the planets is 'x'. The ratio of the escape velocities from the two planets is :

(A) xr (B) $\sqrt{\frac{r}{x}}$

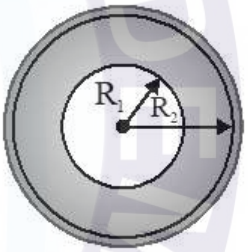
(C) $\sqrt{rx}$ (D) $\sqrt{\frac{x}{r}}$

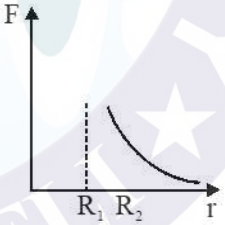
**17.** Two bodies of masses $m_1$ and $m_2$ are initially at rest at infinite distance apart. They are then allowed to move towards each other under mutual gravitational attraction. Their relative velocity of approach at a separation distance r between them is:-

(A) $\left[2G\frac{(m_1 - m_2)}{r}\right]^{1/2}$ (B) $\left[\frac{2G}{r}(m_1 + m_2)\right]^{1/2}$

(C) $\left[\frac{r}{2G(m_1 m_2)}\right]^{1/2}$ (D) $\left[\frac{2G}{r}m_1 m_2\right]^{1/2}$

**18.** A sphere of mass M and radius $R_2$ has a concentric cavity of radius $R_1$ as shown in figure. The force F exerted by the sphere on a particle of mass m located at a distance r from the centre of sphere varies as $(0 \le r \le \infty)$ :




(A)
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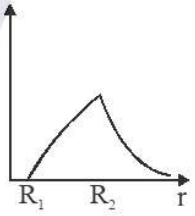
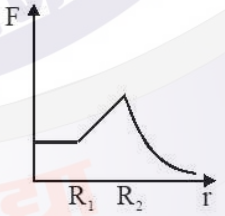
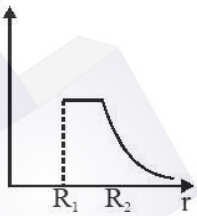



(C)



(D)


**19.** If the earth is a point mass of $6 \times 10^{24}$ kg revolving around the sun at a distance of $1.5 \times 10^{8}$ km and in time $T = 3.14 \times 10^{7}$s, then the angular momentum of the earth around the sun is :-

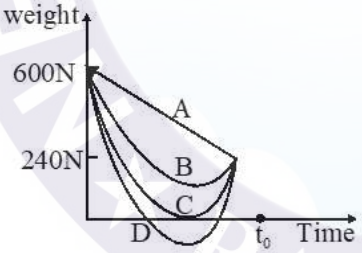
(A) $1.2 \times 10^{18}$ kgm$^2$/s

(B) $1.8 \times 10^{29}$ kgm$^2$/s

(C) $1.5 \times 10^{37}$ kgm$^2$/s

(D) $2.7 \times 10^{40}$ kgm$^2$/s

**20.** Suppose, the acceleration due to gravity at the earth's surface is 10 m/s$^2$ and at the surface of Mars it is 4.0 m/s$^2$. A 60 kg passenger goes from the earth to the mars in a spaceship moving with a constant velocity. Neglect all other objects in the sky. Which part of figure best represents the weight (net gravitational force) of the passenger as a function of time :




(A) A

(B) B

(C) C

(D) D

## SECTION-II : (Maximum Marks: 20)

**This section contains 10 questions Candidates have to attempt any 5 questions out of 10. If more than 5 questions are attempted, then only first 5 attempted questions will be evaluated.**

The answer to each question is a **Numerical Value.**

For each question, enter the correct integer value (In case of non-integer value, the answer should be rounded off to the nearest Integer).

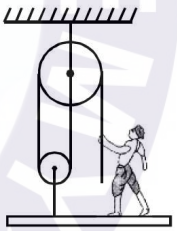
Answer to each question will be evaluated according to the following marking scheme:

*Full Marks* : +4 If correct answer is entered.

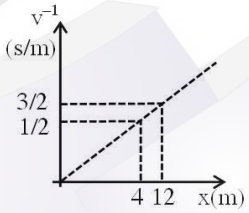
*Zero Marks* : 0 If the question is unanswered.

*Negative Marks* : –1 If wrong answer is entered.

**1.** A man of mass 60 kg is standing on a platform of mass 40 kg as shown in figure then what force man should apply on rope so that he accelerate up with the platform with acceleration of 2 m/s$^2$.

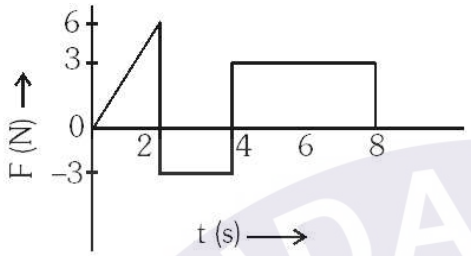


**2.** Graph of 1/v versus x for a particle under motion is shown as, where v is velocity and x is position. The time taken by particle to move from x = 4 to x = 12 m is :-

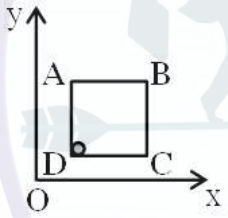



**Space for Rough Work**

**3.** The force 'F' acting on a particle of mass 'm' is indicated by the force-time graph shown below. The change in momentum of the particle over the time interval from zero to 8 s is :-




**4.** A solid sphere of mass 2 kg is resting inside a cube as shown in the figure. The cube is moving with a velocity $\vec{v} = \left(5t\hat{i} + 2t\hat{j}\right)$ m/s. Here t is the time in second. All surface are smooth. The sphere is at rest with respect to the cube. The total force exerted by the sphere on the cube is 13 x Newton, then find x ? (Take g = 10 m/s$^2$)




**5.** The minimum force required to start pushing a body up a rough (frictional coefficient $\mu$) inclined plane is $F_1$ while the minimum force needed to prevent it from sliding down is $F_2$. If the inclined plane makes and angle $\theta$ from the horizontal such that $\tan\theta = 2\mu$ then the ratio $\frac{F_1}{F_2}$ is:-

**6.** The mass of planet is 1/9 of the mass of the earth and its radius is half that of the earth. If a body weight 9 N on the earth. Its weight on the planet would be :- (in N)

**7.** Two planets A and B have the same material density. If the radius of A is twice that of B, then the ratio of the escape velocity $\frac{v_A}{v_B}$ is :-

**8.** Two satellites, A and B, have masses m and 2m respectively. A is in a circular orbit of radius R, and B is in a circular orbit of radius 2R around the earth. The ratio of their kinetic energies, $T_A/T_B$, is:

**9.** Gravitational force between two masses at a distance d apart is 6N. If these masses are taken to moon and kept at a same separation, then the force between them will become :-

**10.** Three equal masses of 1 kg each are placed at the vertices of an equilateral triangle PQR and a mass of 2 kg is placed at the centroid O of the triangle which is at a distance of $\sqrt{2}$ m from each of the vertices of the triangle. The force, in newton, acting on the mass of 2 kg is :-

**Space for Rough Work**

## PART-2 : CHEMISTRY

### SECTION-I : (Maximum Marks: 80)

This section contains **20 questions**. Each question has 4 options for correct answer. Multiple-Choice Questions (MCQs) **Only one option is correct**. For each question, marks will be awarded as follows:

*Full Marks* : +4 If correct answer is selected.

*Zero Marks* : 0 If none of the option is selected.

*Negative Marks* : –1 If wrong option is selected.

**1.** Which of the following process is correct ?

(A) $Ca(NO_3)_2 \xrightarrow{\Delta} CaO + 2NO_2\uparrow + \frac{1}{2}O_2\uparrow$

$\Delta \downarrow$

$Ca + \frac{1}{2}O_2\uparrow$

(B) $2FeSO_4 \xrightarrow{\Delta} Fe_2O_3 + SO_2\uparrow + SO_3\uparrow$

(C) $Ca(HCO_3)_2 \xrightarrow{\Delta} CaCO_3 + CO_2\uparrow + H_2O$

(D) Both (B) and (C)

**2.** Select wrong statement :-

(A) A transition metal ion has more polarizing power than S-block ions of comparable size and charge.

(B) Order of solubility in water is

$AgF > AgCl > AgBr > AgI$

(C) LiCl is soluble in organic solvents

(D) The hydration of ions involves absorption of heat

**3.** Which is set of paramagnetic molecules or ions?

(A) $NO, NO_2, O_2^{2+}, O_2^{2-}$

(B) $O_2^+, O_2^-, NO, NO_2$

(C) $C_2, B_2, O_2, S_2$

(D) $B_2, N_2, O_2, O_2^-$

**4.** The hybridization of the central atom will change when :

(A) $NH_3$ combines with $H^+$

(B) $H_3BO_3$ combines with $OH^-$

(C) $NH_3$ forms $NH_2^-$

(D) $H_2O$ combines with $H^+$

**5.** Which orbitals overlap to form bond in $OF_2$ ?

(A) $sp^3 - 2p$ (B) $sp^2 - 2p$

(C) $sp - 2p$ (D) $p - p$

**6.** Which of the following has highest melting point ?

(A) $SF_6$ (B) NaCl

(C) SiC (D) Xe

**7.** Which of the following molecular orbital has lowest energy for $B_2$ molecule?

(A) $\sigma_{2p_x}$ (B) $\sigma^*_{2p_x}$

(C) $\pi_{2p_y}$ (D) $\pi^*_{2p_y}$

**8.** Select incorrect order :-

(A) $NO_3^- > NO_2^- > NO_2^+$ (N – O bond length)

(B) HI > HF > HBr > HCl (order of melting point)

(C) $CH_4 < CH_3Cl < CH_2Cl_2 < CHCl_3$ (order of dipole moment)

(D) $SbH_3 > NH_3 > AsH_3 > PH_3$ (order of boiling point)

**Space for Rough Work**

**9.** Which of the following does not have coordinate bonds ?

(A) $HNO_2$ (B) $O_3$

(C) $NaBF_4$ (D) $NH_4Cl$

**10.** An element (X) forms compounds of the formula $XCl_3$, $X_2O_5$ and $Ca_3X_2$ but does not for $XCl_5$. Predict element (X) :

(A) B (B) Al (C) N (D) P

**11.** Find enthalpy of neutralisation of $NH_4OH$ and HCN in aqueous solution if enthalpy of ionisation of $NH_4OH$ and HCN are 7 kJ/mol and 8 kJ/mol. also enthalpy of ionisation of $H_2O$ is 57.3 kJ/mole.

(A) $-15$ kJ/mol (B) $-$ 42.3 kJ/mol

(C) +1 kJ/mol (D) 42.3 kJ/mol

**12.** Determine $\Delta U^\circ$ at 300 K for the following reaction using the listed enthalpies of reaction :-

$4CO(g) + 8H_2(g) \longrightarrow 3CH_4(g) + CO_2(g) + 2H_2O(\ell)$

$C(\text{graphite}) + \frac{1}{2} O_2(g) \longrightarrow CO(g);\ \Delta H_1^\circ = -110.5$ kJ

$CO(g) + \frac{1}{2} O_2(g) \longrightarrow CO_2(g);\ \Delta H_2^\circ = -282.9$ kJ

$H_2(g) + \frac{1}{2} O_2(g) \longrightarrow H_2O(\ell);\ \Delta H_3^\circ = -285.8$ kJ

$C(\text{graphite}) + 2H_2(g) \longrightarrow CH_4(g);\ \Delta H_4^\circ = -74.8$ kJ

(A) $-653.5$ kJ (B) $-686.2$ kJ

(C) +747.4 kJ (D) None of these

**13.** Two mole of an ideal gas is heated at constant pressure of one atmosphere from 27°C to 127°C. If $C_{v,m} = 20 + 10^{-2}\ T\ JK^{-1}\ mol^{-1}$, then q and $\Delta U$ for the process are respectively :-

(A) 6362.8 J, 4700 J (B) 3037.2 J, 4700 J

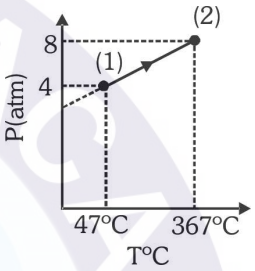
(C) 7062.8 J, 5400 J (D) 3181.4 J, 2350 J

**14.** For a reversible process at T = 300 K, the volume of 2 mole of ideal gas is increased from 1 litre to 10 litre, the $\Delta H$ for isothermal change is :

(A) 11.47 kJ (B) 4.98 kJ

(C) 0 (D) 114.7 kJ

**15.** 2 moles of an ideal diatomic gas is subjected to the following process. Mark the correct option(s). [Given : R = 2 Cal/mol K]




(A) w = 0 (B) q = 4480 Cal

(C) $\Delta H$ = 2240 Cal (D) $\Delta E$ = 1600 Cal

**16.** 2 mole of an ideal gas at 27° C temperature is expanded reversibly from 2 lit to 20 lit. Find entropy change (R = 2 cal/mol K) :-

(A) 92.1 cal $K^{-1}\ mol^{-1}$

(B) 0 cal $K^{-1}\ mol^{-1}$

(C) 4 cal $K^{-1}\ mol^{-1}$

(D) 9.2 cal $K^{-1}\ mol^{-1}$

**17.** In which of the reaction entropy decreases?

(A) $C(s) + O_2(g) \longrightarrow CO_2(g)$

(B) $H_2(g) + I_2(g) \longrightarrow 2HI(g)$

(C) $PCl_5(g) \longrightarrow PCl_3(g) + Cl_2(g)$

(D) $N_2(g) + 3H_2(g) \longrightarrow 2NH_3(g)$

**Space for Rough Work**

**18.** For the reaction takes place at certain temperature

$NH_2COONH_4(s) \rightleftharpoons 2NH_3(g) + CO_2(g)$.

If equilibrium pressure is 3X bar then $\Delta_r G°$ would be :-

(A) $-RT \ln 4 - 3RT \ln X$

(B) $RT \ln 4 - 3RT \ln X$

(C) $-3RT \ln X$

(D) None of these

**19.** $1/2\ A_2 + 3/2B_2 \longrightarrow AB_3$, $\Delta H° = -20$ kJ $S°$ for $A_2$, $B_2$ and $AB_3$ are 60, 40 and 50 $JK^{-1}\ mol^{-1}$ respectively then calculate the temperature at which equilbrium will established :-

(A) 500 K

(B) 200 K

(C) 1000 K

(D) All temp.

**20.** Oxygen gas weighing 64 gm is expanded isothermally and reversibly from 1 atm to 0.25 atm at 30°C. Calculate entropy change (in $JK^{-1}\ mol^{-1}$), assuming the gas to be ideal.

(A) 18 $JK^{-1}\ mol^{-1}$

(B) 23 $JK^{-1}\ mol^{-1}$

(C) 30 $JK^{-1}\ mol^{-1}$

(D) 15 $JK^{-1}\ mol^{-1}$

## SECTION-II : (Maximum Marks: 20)

**This section contains 10 questions Candidates have to attempt any 5 questions out of 10. If more than 5 questions are attempted, then only first 5 attempted questions will be evaluated.**

The answer to each question is a **Numerical Value.**

For each question, enter the correct integer value (In case of non-integer value, the answer should be rounded off to the nearest Integer).

Answer to each question will be evaluated according to the following marking scheme:

*Full Marks* : +4 If correct answer is entered.

*Zero Marks* : 0 If the question is unanswered.

*Negative Marks* : −1 If wrong answer is entered.

**1.** How many of the following will evolve $O_2$ gas upon heating ?

(i) $KMnO_4$ (ii) $Ca(NO_3)_2$

(iii) $Na_2SO_4$ (iv) $K_2CO_3$

(v) $NaNO_3$ (vi) $(NH_4)_2\ Cr_2O_7$

(vii) $NH_4NO_3$ (viii) $CaC_2O_4$

**2.** In how many of the following species axial d-orbitals participate in hybridization ?

$XeF_2$, $CO_2$, $XeF_4$, $H_2CO_3$, $SOF_4$, $SO_4^{2-}$, $NO_3^-$

**3.** How many of the following have one unpaired electron ?

$KO_2$, $O_2[BF_4]$, $K_2O_2$, $O_2$, $N_2$, $NO_2$, $B_2H_6$

**4.** How many of the following are valid formulaes ?

$NCl_5$, $PH_5$, HI, $ClF_7$, $SF_4$, $BF_6^{-3}$, $BH_4^-$, $B(OH)_4^-$

Space for Rough Work

**5.** In the formation of $XeF_5^{\oplus}$ total number of orbitals of Xe involved in Hybridisation is :-

**6.** Standard enthalpies of formation of CO(g), $CO_2(g)$, $N_2O(g)$ and $N_2O_4(g)$ are –110, –393, –81 and 10 kJ/mole, respectively. Find the $\Delta_rH°$ for the reaction :

$N_2O_4(g) + 3CO \longrightarrow N_2O(g) + 3CO_2(g)$

**7.** Calculate enthalpy change for the reaction in Kcal/mol

$H_2(g) + C_2H_4(g) \longrightarrow C_2H_6(g)$

The bond energies are,

H–H ⇒ 110, C—H ⇒ 100, C—C ⇒ 80 &

C=C ⇒ 150 Kcal $mol^{-1}$

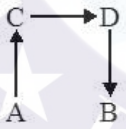
**8.** What is the final temperature (in kelvin) of 0.10 mole monoatomic ideal gas that performs 75 cal of work adiabatically if the initial temperature is 227ºC ? (use R = 2 cal/K-mol)

**9.** For the reaction,

$A_{(g)} + 2B_{(g)} \longrightarrow 2C_{(g)} + 3D_{(g)}$

the value of $\Delta H$ at 27°C is 19.0 KCal. Calculate value of $\Delta U$ for the reaction in Kcal :-

**10.** A process A ⟶ B is difficult to occur directly instead it takes place in three successive steps.




$\Delta S(A \longrightarrow C) = 50$ e.u.

$\Delta S(C \longrightarrow D) = 30$ e.u.

$\Delta S(B \longrightarrow D) = 20$ e.u.

Where e.u. is entropy unit.

Then the entropy change for the process $\Delta S(A \longrightarrow B)$ is :-

## PART-3 : MATHEMATICS

### SECTION-I : (Maximum Marks: 80)

This section contains **20 questions**. Each question has 4 options for correct answer. Multiple-Choice Questions (MCQs) **Only one option is correct**. For each question, marks will be awarded as follows:

*Full Marks* : +4 If correct answer is selected.

*Zero Marks* : 0 If none of the option is selected.

*Negative Marks* : –1 If wrong option is selected.

**1.** If $\phi(x) = f(x) + f(2a - x)$ and $f''(x) > 0$, $a > 0$, $0 \le x \le 2a$, then :-

(A) $\phi(x)$ increases in (a, 2a)

(B) $\phi(x)$ increases in (0, a)

(C) $\phi(x)$ decreases in (a, 2a)

(D) None

**2.** If $f(x) = (x - 1)(x - 2)(x - 3)$ and a = 0, b = 4, then c the using LMVT is $2 \pm \lambda$. Then value of $450\sqrt{2}\,\lambda$ must be:-

(A) 400 (B) 500

(C) 600 (D) 700

**3.** Let $f(x) = \begin{cases} \sin^{-1}\alpha + x^2, & 0 < x < 1 \\ 2x, & x \ge 1 \end{cases}$

f(x) can have a minimum at x = 1 is the value of $\alpha$ is :-

(A) 1

(B) -1

(C) 0

(D) 2

**Space for Rough Work**

**4.** The number of points in $(-\infty, \infty)$, for which $x^2 - x\sin x - \cos x = 0$, is :

(A) 6 (B) 4

(C) 2 (D) 0

**5.** The global maximum value of $f(x) = \log_{10}(4x^3 - 12x^2 + 11x - 3)$, $x \in [2, 3]$, is :–

(A) $-\frac{3}{2}\log_{10}3$

(B) $1 + \log_{10}3$

(C) $\log_{10}3$

(D) $\frac{3}{2}\log_{10}3$

**6.** The height of a right circular cone of maximum volume inscribed in a sphere of diameter a is-

(A) (2/3)a

(B) (3/4)a

(C) (1/3)a

(D) (1/4)a

**7.** The tangent to curve $y_1 = ax^2 + bx + \frac{7}{2}$ at (1, 2) is parallel to normal at point (–2, 2) on curve $y_2 = x^2 + 6x + 10$, then value of (a – 2b) is

(A) 4

(B) – 4

(C) 2

(D) 6

**8.** The length of sub-normal to the curve $xy^n = a^{n+1}$ is constant then value of n is

(A) –2 (B) –1 (C) 0 (D) 2

**9.** The number of critical point(s) of $f(x) = (x - 1)|x - 2| + (x - 2)|x - 1|$ is :-

(A) 2 (B) 1

(C) more than 2 (D) $-\frac{1}{2}$

**10.** If $f(x) = (|x|)^{|\sin x|}$, then $f'\left(-\frac{\pi}{4}\right) =$

(A) $\left(\frac{\pi}{4}\right)^{1/\sqrt{2}}\left(\frac{\sqrt{2}}{2}\log\frac{4}{\pi} - \frac{2\sqrt{2}}{\pi}\right)$

(B) $\left(\frac{\pi}{4}\right)^{1/\sqrt{2}}\left(\frac{\sqrt{2}}{2}\log\frac{4}{\pi} + \frac{2\sqrt{2}}{\pi}\right)$

(C) $\left(\frac{\pi}{4}\right)^{1/\sqrt{2}}\left(\frac{\sqrt{2}}{2}\log\frac{\pi}{4} - \frac{2\sqrt{2}}{\pi}\right)$

(D) $\left(\frac{\pi}{4}\right)^{1/\sqrt{2}}\left(\frac{\sqrt{2}}{2}\log\frac{\pi}{4} + \frac{2\sqrt{2}}{\pi}\right)$

**11.** Let $f(x) = 3^{\alpha x} + 3^{\beta x}$, where $\alpha \neq \beta$ and $3f'(x)\log_3 e = 2f(x) + f''(x).(\log_3 e)^2$ for all x. Then the value of $\alpha + \beta$ is :

(A) 3 (B) 2 (C) –3 (D) 6

**12.** If $y = \tan^{-1}\left(\frac{\log(e/x^2)}{\log(ex^2)}\right) + \tan^{-1}\left(\frac{3 + 2\log x}{1 - 6\log x}\right)$, then $\frac{d^2y}{dx^2}$ is

(A) 2 (B) 1 (C) 0 (D) –1

**13.** Derivative of $\cot^{-1}\left(\frac{2\sqrt{1+x^2} - 5\sqrt{1-x^2}}{5\sqrt{1+x^2} + 2\sqrt{1-x^2}}\right)$ with respect to $\cos^{-1}\sqrt{1-x^4}$ is equal to :-

(A) 1 (B) –1 (C) 1/2 (D) –1/2

**Space for Rough Work**

**14.** If $y = \sqrt{(x-\sin x)+\sqrt{(x-\sin x)+\sqrt{(x-\sin x)+\ldots\ldots\infty}}}$, then value of $\left|\left(\frac{dx}{dy}\right)^2_{x=\frac{\pi}{2}} - 2\pi\right|$ is :

(A) 1 (B) 2

(C) 3 (D) 4

**15.** In a triangle tanA + tan B + tan C = 6 and tan A tan B = 2, then the values of tan A, tan B and tan C are :-

(A) 1,2,3

(B) 3,2,1

(C) 1,2,0

(D) None of these

**16.** The value of

$$\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{7\pi}{14}\sin\frac{9\pi}{14}\sin\frac{11\pi}{14}\sin\frac{13\pi}{14}$$

is equal to

(A) $\frac{1}{8}$

(B) $\frac{1}{16}$

(C) $\frac{1}{32}$

(D) $\frac{1}{64}$

**17.** Let a = cos A + cos B − cos (A + B)

b = 4 sin $\frac{A}{2}$ sin $\frac{B}{2}$ cos $\left(\frac{A+B}{2}\right)$.

Then a − b equal to :-

(A) 1 (B) 0 (C) −1 (D) None

**18.** If $\alpha, \beta, \gamma$ and $\delta$ are the solutions of the equation $\tan\left(\theta+\frac{\pi}{4}\right)$ =3 tan 3$\theta$, no two of which have equal tangents, then the value of

tan $\alpha$ + tan $\beta$ + tan $\gamma$ + tan $\delta$ is :

(A) 1 (B) -1

(C) 2 (D) 0

**19.** If $\tan^2\left(\frac{\pi}{16}\right)+\tan^2\left(\frac{2\pi}{16}\right)+\tan^2\left(\frac{3\pi}{16}\right)+\ldots\ldots$

$\ldots\ldots\ldots+\tan^2\left(\frac{7\pi}{16}\right)=\lambda$ & if $x^y + y^x = \lambda$, then the value of $(x+y)^2$ must be

(A) 35 (B) 1225

(C) 225 (D) 2

**20.** Let x = sin 1°, then the value of the expression

$$\frac{1}{\cos 0^\circ.\cos 1^\circ}+\frac{1}{\cos 1^\circ.\cos 2^\circ}+\frac{1}{\cos 2^\circ.\cos 3^\circ}+\ldots+\frac{1}{\cos 44^\circ.\cos 45^\circ}$$

is equal to

(A) x

(B) $\frac{1}{x}$

(C) $\frac{\sqrt{2}}{x}$

(D) $\frac{x}{\sqrt{2}}$



Space for Rough Work

## SECTION-II : (Maximum Marks: 20)

**This section contains 10 questions Candidates have to attempt any 5 questions out of 10. If more than 5 questions are attempted, then only first 5 attempted questions will be evaluated.**

The answer to each question is a **Numerical Value.**

For each question, enter the correct integer value (In case of non-integer value, the answer should be rounded off to the nearest Integer).

Answer to each question will be evaluated according to the following marking scheme:

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*Zero Marks* : 0 If the question is unanswered.

*Negative Marks* : −1 If wrong answer is entered.

**1.** If the function f given by

$f(x) = x^3 - 3(a-2)x^2 + 3ax + 7$, for some $a \in R$

is increasing in $(0, 1]$ and decreasing in $[1, 5)$, then a root of the equation, $\frac{f(x)-14}{(x-1)^2} = 0 (x \neq 1)$ is :

**2.** The sum of the maximum and minimum values of the function $f(x) = |5x - 7| + [x^2 + 2x]$ is the interval $\left[\frac{5}{4}, 2\right]$, where [t] is the greatest integer $\leq t$ is :

**3.** If 'R' is the least value of 'a' such that the function $f(x) = x^2 + ax + 1$ is increasing on [1, 2] and 'S' is the greatest value of 'a' such that the function $f(x) = x^2 + ax + 1$ is decreasing on [1, 2], then the value of $|R - S|$ is :

**4.** If $y^{1/4} + y^{-1/4} = 2x$, and

$(x^2 - 1)\frac{d^2y}{dx^2} + \alpha x\frac{dy}{dx} + \beta y = 0$, then $|\alpha - \beta|$ is equal to _____.

**5.** If $y = \sum_{k=1}^{6} k\cos^{-1}\left\{\frac{3}{5}\cos kx - \frac{4}{5}\sin kx\right\}$, then $\frac{dy}{dx}$ at $x = 0$ is_____.

**6.** If $K = \frac{\cos^2 1^\circ - \cos^2 2^\circ}{2\sin 3^\circ \sin 1^\circ}$, then $2K = ?$

**7.** If $\theta = \frac{\pi}{4n}$ then value of

$\tan\theta \tan 2\theta$ _____ $\tan(2n-2)\theta \tan(2n-1)\theta$

is :-

**8.** If $\tan\left(\frac{\pi}{4} + \theta\right) + \tan\left(\frac{\pi}{4} - \theta\right) = k \sec 2\theta$ then k is equal to :-

**9.** Let f(x) be a cubic polynomial with $f(1) = -10$, $f(-1) = 6$, and has a local minima at $x = 1$, and $f'(x)$ has a local minima at $x = -1$. Then $f(3)$ is equal to _____.

**10.** Cosine of the angle of intersection of curves $y = 3^{x-1}\log x$ and $y = x^x - 1$ is

**Space for Rough Work**